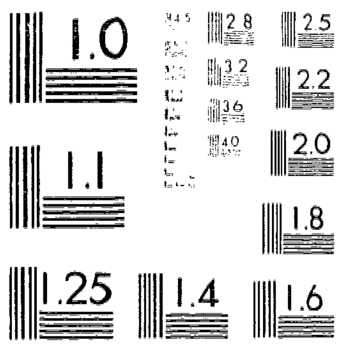
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ABSTRACT

The purpose of this study was to investigate the relationship between a student's confidence in his computational procedures for each of the four basic arithmetic operations and the student's achievement on computation problems. All of the students in grades 5 through 8 in one school system (a total of 6186 students) were given a questionnaire to determine their algorithmic confidence and a computational test for each of the four basic arithmetic operations on whole numbers. Addition and multiplication tables accompanied the test. Data on 5440 responses were used in the analyses. "Low achievers" on a particular computation test were defined as those students scoring more than one standard deviation below the mean of that test. There were a total of 267 low achievers on the addition test, 734 on the subtraction test, 735 on the multiplication test, and 985 on the division test. Of these low achievers, 226 expressed high algorithmic confidence in addition, 576 in subtraction, 513 in multiplication and 440 in division. The investigator concluded that for each arithmetic operation there were a substantial number of low achievers who expressed high algorithmic confidence. (DT)

Mathematics Education Diagnostic and Instructional Centre.

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MEDIC REPORT 2-75

A Comparison of Students' Achievement in Arithmetic with their Algorithmic Confidence

by
Irene Douglas MacKay

Report from the
Richmond Project (ORACLE)

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CONTENTS

Chapter 1

INTRODUCTION

Many students who are referred to the Mathematics Diagnostic and Instructional Clinic (MEDIC) at the University of British Columbia for remediation are confident that their computational procedures are correct. They are confident even though they are usually unable to obtain a correct answer. It is felt that remediation may be hampered by the fact that a student believes his computational procedures are correct, when, in fact, he is unable to compute accurately. It was decided that there is a need to study the relationship between a student's algorithmic confidence in performing each of the four basic arithmetic operations and the student's achievement.

The District Superintendent for the Richmond School Board, Mr. C. Holob, was contacted by letter (Appendix I) asking permission to gather some achievement and algorithmic confidence data on students enrolled in Grades 5 through 8 in the Richmond schools. The proposed study was also described in this letter. In reply, the District Superintendent expressed the willingness of the Richmond schools to participate in the project. A notice (Appendix II) was sent to all elementary principals and teachers of Grades 5, 6, and 7 by Mr. Holob regarding the MEDIC project.

The testing project was carried out with all students of Grades 5, 6, 7, and 8 in the Richmond District. The following table gives the number of students according to grade level in the district.

Table 1

Enrolment in Grades 5-8 December 1974					
Grade	5	6	7	8	Total
Enrolment	1472	1577	1566	1571	6186

Test booklets (Appendix III) were prepared according to the following format. The first page was designed to collect personal data on each student--name, grade, division, school, age, date of birth, sex--and their algorithmic confidence. To determine a student's algorithmic confidence in addition, for example, the student was asked the following question. How sure are you that your way of ADDING is correct? The student had a choice of five replies and was asked to respond to one by putting an X through one of the letters a, b, c, d, or e. These were their choices:

a. I'm positive that my way is correct.

b. I'm pretty sure that my way is correct.

c. I don't know if my way is correct or not.

d. I'm pretty sure my way is wrong.

e. I'm positive my way is wrong.

The same questions were asked for the other three arithmetic operations--subtraction, multiplication and division.

The computational test consisted of four sub-tests. That is, there was a test in each of the four basic arithmetic operations of whole numbers. Accompanying each test was a sheet containing the addition and multiplication tables (Appendix IV). These tables were made available in an effort to eliminate errors

in basic number facts so that the emphasis would be more directly placed on the student's computational procedures. The test was so designed that it included different categories of question types in each of the sub-tests. The items for each test were selected according to a diagnosis form for intermediate grades (Appendix V). The form or check-list is structured in an heirarchial order of difficulty. That is, the check-list is arranged in order of harder to easier computations. For instance, in the addition of whole numbers the order of difficulty is as follows:

1. three-digit numbers with regrouping.
2. two-digit numbers with regrouping
3. single column with regrouping.
4. single column with no regrouping.

Generally three test items were constructed corresponding to each of the four levels of algorithmic difficulty. Other considerations were observed to maintain variety in the question types. For example, in subtraction the vertical and horizontal form were included. Different positions of zero in the minuend and subtrahend were used. The following table indicates the types of questions according to the check-list and their distribution on the sub-tests.

Table II

Distribution of Addition Items According to Type

	Items	a	b	c	d	e	f	g	h	i	j	k	l
	1			✓			✓			✓			
Types	2				✓			✓					✓
	3	✓							✓			✓	
	4		✓			✓					✓		

The check-list for subtraction is:

1. two consecutive 0's in the minuend.
2. one 0 in the minuend.
3. with regrouping.
4. no regrouping.

Table III

Distribution of Subtraction Items According to Type

	Items	a	b	c	d	e	f	g	h	i	j	k	l
Types	1	✓			✓		✓				✓		
	2					✓		✓					
	3			✓						✓		✓	
	4		✓						✓				✓

Multiplication check-list:

1. Three-digit multiplier.
2. two-digit multiplier.
3. one-digit multiplier; with regrouping.
4. one-digit multiplier with no regrouping.

Table IV

Distribution of Multiplication Items According to Type

	Items	a	b	c	d	e	f	g	h	i	j	k	l
Types	1	✓	✓					✓					
	2				✓					✓			✓
	3			✓		✓	✓						
	4								✓		✓	✓	

Division check-list:

1. Zero in the quotient.
2. Two-digit divisors without or with remainder.
3. One-digit divisor with remainder.

4. One-digit divisor with no remainder.

Table V

Distribution of Division Items According to Type

Types / Items	a	b	c	d	e	f	g	h
1			✓				✓	
2				✓	✓			
3	✓	✓						
4						✓		✓

Previous to the Richmond project a pilot study was conducted about the middle of January, 1975 at the University Hill Elementary School. Sixty students in grades 5 and 6 were involved. The reason for the pilot study was to obtain an estimate of the time required for each test and to find out if there were any problems with the test. The time required was approximately forty minutes for 90% of the students. Following the pilot study, the tests were administered to the students of Richmond District the first week in February, 1975.

Classroom teachers conducted the testing with their students. Teachers were previously instructed that the tests were to be administered according to the following procedures. Firstly, the students were to be shown how to use the addition and multiplication tables which were provided. Secondly, teachers were to explain the structure of the tests to the students. Thirdly, the students were to be told that there was no time limit on the tests.

Following the testing about 5700 responses were collected of which 5440 were used. Table VI shows the distribution of testing according to grade and sex.

Table IV

Grade	5	6	7	8	Total
Boys	705	728	747	606	2786
Girls	655	746	664	589	2654
Total	1360	1474	1411	1195	5440

Two hundred sixty tests were discarded because some of the personal information requested on each student--name, grade, age, date of birth--was missing or else the student did not complete the assessment of his algorithmic confidence in computational procedures. The tests were marked for accuracy by undergraduate students in February and March, 1975. All numerical data were collected and key punched for computer analysis about mid April, 1975. During June and July, 1975 all student errors will be examined and coded according to a category system which had previously been developed.

The data will be analyzed for two different purposes: firstly, to examine the relationships between confidence and performance over operations, grades, sexes, ages, and confidence and performance levels; secondly, to examine the relationships between different error types over operations, grades, sexes, ages, and confidence and performance levels.

The objective of this paper is to discuss the first of these two purposes.

Chapter 2

RATIONALE

Relative to the various changes and activities taking place in the elementary arithmetic curriculum and certain areas of the instructional program, the area of diagnosis and remediation has been quite static. Early work in diagnosis in arithmetic errors was mostly limited to determining the kinds and frequency of errors in computational skills. Later, concerns were slanted to growth in meanings and understandings basic to the computational process.

More recent concerns in diagnosis are the complex relationship between growth in arithmetic development and affective factors such as anxiety, motivation, and attitude. Ramon Ross (1964) reporting on the twenty case studies carried out with sixth and seventh grade students revealed a great deal of disparity between actual achievement and expected achievement in elementary school mathematics. He suggested that sixty-three percent of the causes of underachievement identified by classroom teachers were of an emotional nature, involving lack of interest, home or school maladjustment, short attention span, or limited initiative. It would appear from this study that arithmetic underachievement is a complex and multiple-factored disability.

John W. Wilson (1967) also expressed a similar concern in diagnosing the cause of underachievement in elementary school mathematics when he states,

> It has become increasingly apparent in our work with individual children...that underachievement... in mathematics...is far from being of one kind... Of several children with the same degree of general underachievement in mathematics, each has unique symptomatic patterns of that underachievement.

The fact that Wilson recognized that each student has "unique symptomatic patterns of underachievement" suggests the complexity of the nature of underachievement as well as the complexity of diagnosing the cause and the method of remediation. In other words, Wilson and Ross underline the complexity of both diagnosing the cause of a student's difficulty in mathematical operations as well as the difficulty in correcting or remediating the problem. The question to be answered is how can you successfully prescribe treatment or remediation without knowing the root of the difficulty? The root of the difficulty that came to the attention of the Mathematics Education Diagnostic and Instructional Centre (MEDIC) at the University of British Columbia was the fact that some students who were referred for remedial help and who were unsuccessful in their daily computations had high confidence that their computational procedures were correct.

The question to be argued in this paper is--how does a student's confidence in his computational procedures affect the subsequent success of remediation? An extensive search of the literature has failed to produce an answer to this question.

To date, in the remediation process, the remediator has tried to build confidence in the student but maybe the remediator is taking the wrong approach. It may be that it is necessary to extinguish a student's computational confidence in his incorrect algorithm before the actual remediation takes place.

Common expectations are held that low achievers have low confidence which needs to be improved. On the contrary, it could be that low achievers have high confidence which has to be extinguished before effective remediation can occur. As already mentioned it has been noticed that among low achievers referred to MEDIC, there is a number of students who are confident that they know how to perform the arithmetic algorithms. But the interesting question is, why do low achievers express algorithmic confidence in their computational procedures when they constantly get most of their exercises wrong? Does a student's expression of confidence in his work stem from a naturally confident personality? It may be the result of positive values taught in the home. These positive values may give rise to a confident, positive outlook on the various activities of life. That is, from an early age, these values, once instilled, may be reflected in a student's personality by feelings of confidence in whatever he does. Woodruff (1962) seems to suggest this when he says,

> Over the years we gradually develop well established feelings about things; and these feelings, based on their values, show up in the way we react toward things. The feelings become inseparably interwoven with the mental picture.

But this argument does not hold because remediators are aware from working with the low achiever that he is not confident in all of his daily activities. Why does he have algorithmic confidence then?

Is it because the low achiever thinks he understands the concept when it is taught by the teacher but in reality he does not? Brownell (1944), in addressing lack of understanding in

students says,

> ...most errors in mathematics are the result not of imperfectly learned symbols, but of incomplete understandings.

Evidence of lack of understanding by the low achiever is apparent in the tests. For example, in using the subtraction algorithm, it was apparent that one student's understanding of the subtraction operation meant to literally take away or remove the subtrahend from the minuend. This is an example of his work:

$$\begin{array}{r} 7749 \\ -7340 \\ \hline 7749 \end{array} \qquad 670 - 97 = 670$$

The student was consistent throughout the subtraction test in literally removing the subtrahend. Another example of lack of understanding by the low achiever is evident in this division exercise.

$$\begin{array}{r} 1001 \\ 5\overline{)7005} \\ \underline{5}\;\;\;5 \\ 2\;\;\;\;\; \end{array} \qquad \text{R.2}$$

It would seem that the student worked from right to left. $5 \div 5 = 1$; $0 \div 5 = 0$; $0 \div 5 = 0$; $7 \div 5 = 1$ and a remainder of 2. This particular student did all of his division exercises from right to left. He expressed a high confidence level of (5). Probably he was confident he knew how to use the division algorithm because he used the other three arithmetic operations--addition, subtraction, and multiplication by beginning the operation at the right. It worked for these operations and lack of understanding led him to beleive it would also work for the division algorithm.

However, the student would still get his exercises marked

wrong. Why is he still confident that his computational procedures are right? Maybe he is using a defense mechanism. That is, he may be protecting his pride and soothing his ego by not admitting to himself that he cannot do his computations. Using a defense mechanism is an attempt by the individual to defend himself against feelings of inferiority occasioned by his failure to do his arithmetic computations. By not admitting to himself that he is incapable of doing arithmetic computations he minimizes his failure to himself. (Loree: 1970) This might be the reason a low achiever indicated algorithmic confidence when his computational procedures are incorrect.

But this paper argues that the main reason for algorithmic confidence stems from random reinforcement.

Skinner, who has been responsible for the concept of reinforcement, noted that some responses occur without any particular stimulus at all. These emitted responses he calls operants. Psychologists before Skinner recognized spontaneous or random responses, but they believed that such responses were caused by some unknown or unidentifiable stimulus. Skinner believes that operants simply occur and that the stimulus conditions are irrelevant to the use and understanding of operant behavior. For Skinner, the fact that the operant be reinforced is important. He believes that if the operant is reinforced the probability of that operant occuring again is increased. What is really important for Skinner is the reinforcement the subject gets after the operant or response is made. This reaction shapes the chances of the student giving this operant response again or of his

giving a similar response in the same class of responses.

Responses, then, are the most important aspect of operant learning, and the way they are reinforced determines most of the qualities of that learning. One of the first discoveries that Skinner made was that operants can be shaped without rewarding or reinforcing every response. He realized that it is not necessary to reinforce after every desired response but only intermittently during the course of several such responses. This realization led Skinner to study two basic patterns of reinforcement. In the first, interval reinforcement, a reward is given on a fixed interval of time--say, every three minutes. In the second, ratio reinforcement, a reward is given after a fixed ratio of responses--say, after every ten or fifteen responses have occurred. Oddly enough, Skinner found that the less frequent the reinforcement on a ratio schedule, the more rapid the response. That is, the animal behaved as if he knew that the faster he responded the faster he would be reinforced.

Both fixed interval and fixed ratio reinforcement schedules are characterized by a pause in response just after reinforcement. Animals seem to know that the responses made just after a reinforcement will never result in another immediate reinforcement. These pauses do not occur if the reinforcement schedule is made random. If the time interval size is varied at random, there is always a chance that the next response after reinforcement could result in another reinforcement, and the animal does not pause.

Is this strange animal behavior reflected in human behavior? It certainly is. Consider for a moment a Los Vegas slot machine player who gets an occasional or random payoff. He plays

vigorously because he does not know at what moment the next payoff will come. But he keeps playing because he is confident that the payoff will come.

What does a Los Vegas slot machine player's confidence have to do with a student's algorithmic confidence? The student's algorithmic confidence is reinforced the same way that the slot machine player's confidence is reinforced. That is by random reinforcement, which according to Skinner is the best reinforcement and the hardest to extinguish. The Richmond Study shows that students using an incorrect arithmetic algorithm can get some exercises correct. The algorithm may not work for many exercises but every now and then it will produce a correct answer. Therefore, the student gets random reinforcement which mades him feel confident that the algorithm he is using is correct. The following examples will show that an incorrect algorithm will work in some cases but not in others. In the example:

$$\begin{array}{r} 299 \\ \cancel{3}\cancel{0}\cancel{0}17 \\ -\ \underline{28\ 9} \end{array}$$

the student begins renaming by crossing out the 3 and writing 2. Crosses out the 0 and writes 9. Crosses out the next 0 to the right and writes 9. He finishes renaming by putting a 1 in front of the 7. This is correct. His algorithm works for this exercise but using the same algorithm in the following example it does not.

$$\begin{array}{r} 1299 \\ \cancel{2}\cancel{3}\cancel{0}\cancel{0} \\ -\ \underline{857} \end{array}$$

The student begins his renaming by stroking out the 2 at the left, then the 3 and writing 2 above it. 0 is crossed out with 9 written above and also the last 0 on the right is crossed out

with a 9 written above it. This is incorrect. The student's algorithm does not work if a 0 comes at the end of the minuend. However, using this algorithm the student will sometimes get some of his exercises correct.

In multiplication, a student doing the following exercise gets a correct answer using the multiplication algorithm as he knows it.

```
 9056
x   3
27168
```

but when asked to find the product of

```
     9056
    x  23
1811227168
```

his answer is incorrect. But the algorithm he is using does work for some of his exercises.

Using an incorrect algorithm, the low achiever may occasionally get an exercise correct. This gives the student random reinforcement. According to Skinner, he is getting the best reinforcement to keep him at work. He does not question whether or not his computational procedures are correct. Why should they be wrong when every once in awhile he gets a correct answer? This keeps him confident that his computational procedures are correct. What does this mean to the remediator? It means that before a remediator can help a student correct his computational procedures that the student's algorithmic confidence must first be extinguished.

Chapter 3

METHOD AND RESULTS

For the Richmond Study, a low achiever in a given operation and grade is defined as one whose score is more than one standard deviation below the mean. Performance distribution for each grade and operation are far from normal distributions. In fact, they are highly skewed positively. But it is felt this commonly accepted approach is more realistic than either choosing a lower fixed percentage of the population or a lower performance level of the population.

Pupil scores, in subtraction and multiplication, ranged from 0 to 12. In division, the range is from 0 to 8 but in addition scores ranged from 1 to 12. The scores of 0 in addition were eliminated because the addition test appeared on the back of the subtraction test. It was assumed that all students who scored 0 on the addition test did not attempt the test.

The tables that follow show the upper-limit for low achievers which was calculated for each operation and at each grade level.

Table I

Grade 5: Upper Limit for Low Achievers

Operations:	+	-	x	÷
Mean	11.0375	10.0162	9.0853	4.0949
Standard Deviation	1.7137	2.4662	2.8721	2.7411
Upper-limit	9.3238	7.5500	6.2132	1.3538

Table II

Grade 6: Upper-Limit for Low Achievers

Operations	+	-	x	÷
Mean	11.2205	10.4579	10.0468	5.2682
S.D.	1.5916	2.0370	2.1749	2.4523
Upper-limit	9.6289	8.4209	7.8719	2.8259

Table III

Grade 7: Upper-Limit for Low Achievers

Operations	+	-	x	÷
Mean	11.3196	10.6641	10.5103	5.8944
S.D.	1.6323	1.8817	1.8855	2.2706
Upper-limit	9.6873	8.7824	8.6248	3.6238

Table IV

Grade 8: Upper-Limit for Low Achievers

Operations	+	-	x	÷
Mean	11.4050	10.8326	10.7272	6.0410
S.D.	1.4412	1.6005	1.6188	2.0630
Upper-limit	9.9638	9.2321	9.1084	3.9773

The actual scores used to categorize the low achiever for each operation and at each grade level is shown in the following table.

Table V

Upper-Limit Used to Determine the Low Achiever

Operations		+	-	x	÷
Grades	5	9	7	6	1
	6	9	8	7	2
	7	9	8	8	3
	8	9	9	9	3

Using these scores as an upper-limit for the low achiever, the next concern was to find the algorithmic confidence by operation and grade level. The following tables give the confidence values from low to high by the numbers 1, 2, 3, 4, and 5. Listed under the confidence values are the corresponding frequency of errors.

Table VI

Addition-Grades 5-8

Frequence \ Confidence		1	2	3	4	5
Grades	5	2	0	24	51	35
	6	1	1	5	26	41
	7	0	1	3	21	24
	8	1	0	3	15	13
Total		4	2	35	113	113

Table VII

Subtraction-Grades 5-8

Frequency \ Confidence		1	2	3	4	5
Grades	5	4	6	58	80	26
	6	3	2	49	112	41
	7	0	4	15	101	50
	8	2	2	13	86	80
Total		9	14	135	379	197

Table VIII

Multiplication-Grades 5-8

Frequency \ Confidence		1	2	3	4	5
Grades	5	11	19	78	97	31
	6	3	8	39	74	32
	7	1	4	25	77	47
	8	0	2	32	92	63
Total		15	33	174	340	173

Table IX

Division-Grades 5-8

Frequency \ Confidence		1	2	3	4	5
Grades	5	44	47	138	85	20
	6	23	26	96	84	21
	7	12	17	68	113	28
	8	8	15	51	69	20
Total		87	105	353	351	89

This study regards the low achiever with high algorithmic confidence to be the number of students in columns 4 and 5. Following is a table showing the number of students in this category.

Table X

Number of Low Achievers with High Algorithmic Confidence

Operation:		Addition			Subtraction			Multiplication			Division			
Confidence		4	5	Sub-total	4	5	Sub-total	4	5	Sub-total	4	5	Sub-total	
Grades	5	51	35	86	80	20	106	97	31	128	85	20	105	425
	6	26	41	67	112	41	153	74	32	106	84	21	105	431
	7	21	24	45	101	50	151	77	47	124	113	23	141	461
	8	15	13	28	86	80	166	92	63	155	69	20	89	438
Sub-Total				226			576			513			440	1755

Table X confirms that there is a substantial number of low achievers in Grades 5 to 8 who have high algorithmic confidence. The sub-totals for confidence levels 4 and 5 show the number of students who expressed algorithmic confidence in each of the arithmetic operations--addition, subtraction, multiplication,

and division. However, it must be noted that in the total number of algorithmic confidences expressed that there is an overlap. The number 1755 does not represent the total number of individual students expressing high algorithmic confidence. One low achiever might indicate a high confidence level in all four basic operations. Therefore, the total number of low achievers with high algorithmic confidence indicated in the table would appear to be four rather than one. Nevertheless, the evidence is clear that for each arithmetic operation there is a substantial number of low achievers who express high algorithmic confidence in each operation.

For purposes of comparison Table XI shows the number of low achievers with low algorithmic confidence.

Table XI

Number of Low Achievers with Low Algorithmic Confidence

Operation:	Addition			Subtraction			Multiplication			Division			
Confidence	1	2	Sub-total	1	2	Sub-total	1	2	Sub-total	1	2	Sub-total	Total
5	2	0	2	4	6	10	11	19	30	44	47	91	133
6	1	1	2	3	2	5	3	8	11	23	26	49	67
7	0	1	1	0	4	4	1	4	5	12	17	29	39
8	1	0	1	2	2	4	0	2	2	8	15	23	30
Sub-total			6			23			48			192	269

By comparing Table XI with Table X, it is quite evident that low achievers with high algorithmic confidence certainly outnumber low achievers with low algorithmic confidence. Also in Table XI, the number 269 does not represent the total number of individual students expressing low algorithmic confidence. As

explained for Table X, one low achiever might indicate a low confidence level for all four basic operations. The table, then, would show a total of four instead of one. This, however, is true for both tables.

The results of this study are conclusive. As suspected, there are a number of students who have algorithmic confidence, yet their computational procedures are incorrect. Table X shows that out of 5440 students from Grades 5 to 8 in the Richmond District 1755 low achievers express high algorithmic confidence in their computational procedures.

Chapter IV

Conclusion and Implications for Further Study

This paper began by describing why it was felt necessary to do this study. It was recognized by remediators that some low achievers had high algorithmic confidence even though their computational procedures were incorrect. Permission was given by the Richmond School Board to conduct the study in that area with all students of Grades 5 to 8. A total of 5440 responses to algorithmic confidence questions and test items were used.

The purpose of this study was to establish the existence of a population of low and high confidence students with low and high algorithmic confidence in their computational procedures. The results indicated, as suspected, that there is such a population and, in fact, low achievers with high algorithmic confidence are in the majority. It is also established, as expected, that there is a different distribution of confidence for each grade level as well as each confidence level. This could suggest that any study done along similar lines of this study could be losing information if it is assumed that student characteristics are uniform across grades or operations.

For this study the low achiever is identified as one whose score is more than one standard deviation below the mean. Tables show how the upper-limit for low achievers was calculated for each operation and at each grade level. It is argued in this paper that the low achiever has high algorithmic confidence in his computational procedures, because of random reinforcement. The student gets random reinforcement when once in awhile an incorrect algorithm gives a correct answer. He, therefore,

assumes that the alorithm he is using is correct. Skinner, in his research work found that random reinforcement is the best reinforcement to keep a subject vigorously at work.

Tables are supplied to indicate that a population of low achievers with high algorithmic confidence does exist in grade levels 5 to 8. Many implications for remediators are raised as a result of this study. When the entire study is completed, teachers, remediators, textbook writers, and computer programmers should have an entirely new challenge in diagnosing and remediating the low achiever.

The results of this study will have important implications to classroom teachers and remediators. Teachers and remediators will be more aware that low achievers have high algorithmic confidence in their computational procedures. It is important that teachers have a systematic method to diagnose and code student's errors. It is evident from this study that a student's algorithmic confidence does have to be extinguished before remediation can take place.

For textbook writers, a change in format and design to meet the needs of low achievers in this category is necessary. The need to be more specific in the writing of behavioral objectives in lesson preparation will help to eliminate learning gaps for these students. Programmed learning, with its step-by-step approach should also assist the low achiever to acquire the correct algorithmic procedure.

The writing of a diagnostic computer program on the basis of all the incorrect answers and subsequent coding of errors for

each question is another means to help correct this problem. A computer program, whereby given the data of the survey, the program can take each question and list all the incorrect answers and corresponding codes. The program could be implemented in such a way that it can diagnose all the errors students perform. A computer remediation program can then be written to do the remediation.

These are but a few of the many implications for educators that result from this study.

Appendix I

January 3, 1975

Mr. C. Holob
District Superintendent of Schools
Richmond School District No. 38
689 No. 3 road
Richmond, B. C.

Dear Mr. Holob:

I am writing to seek your permission to gather some data on students enrolled in Grades 5 through 8 in the Richmond schools. Specifically, I need approximately one hour of class time, preferably divided into two half hour periods on successive days in which to gather some data on the arithmetic skills of these students.

During these two periods the students will be asked to solve addition, subtraction, multiplication, and division examples with whole numbers. In addition, they will be asked to express the degree of confidence they have in their ability to perform these operations.

The data obtained will be useful to me in several ways. As director of the Mathematics Education Clinic at UBC, I need data on the types and frequency of student errors in arithmetic skills. In particular, we have some evidence which indicates that students who are unable to compute correctly still express a relatively high degree of confidence in their ability to compute. If this proves to be the case in a large scale study, it will have important ramifications for remedial work.

For your information I have enclosed a preliminary copy of each of the tests I intend to use. The final version will be printed and will ask for additional information from the student such as age and grade level.

If you are agreeable to this proposal, I would appreciate receiving the following information:

1) a list of the names of your schools where there are grade 5, 6, 7 or 8 classes together with the enrollment in these classes;

...../2

Mr. C. Holob

January 3, 1975

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2) dates when the testing might best be done; (For my purposes late January or early February would be most suitable.)

3) the name of a contact person in your district to act as liaison between your district and me.

Thank you for considering my request.

Sincerely,

David Robitaille
Assistant Professor
Mathematics Education

DR/kl

c.c. Mr. R. Campbell
Encl.

Appendix II

BOARD OF SCHOOL TRUSTEES
SCHOOL DISTRICT NO. 38 (RICHMOND)
689 NO. 3 ROAD, RICHMOND, B.C.
TELEPHONE 278-9521

24-1-75-11

TO *All Elementary Principals and Teachers of Grades 5, 6 & 7.*
FROM *C. Holob, District Superintendent of Schools.*

RE: U.B.C. SURVEY OF MATH SKILLS

Permission has been granted to Dr. David Robitaille to gather data on Richmond students enrolled in Grades 5 through 8. Specifically, Dr. Robitaille seeks data on the types and frequency of student errors in arithmetic skills.

Testing material and teacher instructions, in sufficient number for all students in Grades 5, 6 and 7 will be delivered to schools on January 30/31. The test, requiring approximately 40 minutes should be given early in the week of February 3 with returns to the office of the Elementary Supervisor on or before Friday, February 7, 1975.

The data gathered from this testing programme will assist Dr. Robitaille and his staff to improve their remediation work at the University's Math Clinic. Thank-you, on their behalf, for your assistance with this survey.

C. Holob,
District Superintendent of Schools.

CH:tb

Appendix III

NAME: ______________________ GRADE: ______ DIVISION: ______

SCHOOL: ______________________

AGE: ______________ DATE OF BIRTH: ______________ BOY GIRL (circle one)

For each question, put an X through one of the letters a, b, c, d, or e.

1. How sure are you that your way of ADDING is correct?
   (a) I'm positive that my way is correct.
   (b) I'm pretty sure that my way is correct.
   (c) I don't know if my way is correct or not.
   (d) I'm pretty sure my way is wrong.
   (e) I'm positive my way is wrong.

2. How sure are you that your way of SUBTRACTING is correct?
   (a) I'm positive that my way is correct.
   (b) I'm pretty sure that my way is correct.
   (c) I don't know if my way is correct or not.
   (d) I'm pretty sure that my way is wrong.
   (e) I'm positive that my way is wrong.

3. How sure are you that your way of MULTIPLYING is correct?
   (a) I'm positive that my way is correct.
   (b) I'm pretty sure that my way is correct.
   (c) I don't know if my way is correct or not.
   (d) I'm pretty sure that my way is wrong.
   (e) I'm positive that my way is wrong.

4. How sure are you that your way of DIVIDING is correct?
   (a) I'm positive that my way is correct.
   (b) I'm pretty sure that my way is correct.
   (c) I don't know if my way is correct or not.
   (d) I'm pretty sure that my way is wrong.
   (e) I'm positive that my way is wrong.

SUBTRACTION

(Show all your work in the space provided.)

a.	b.	c.	d.
$\begin{array}{r} 23007 \\ -\ 9739 \\ \hline \end{array}$	$\begin{array}{r} 7749 \\ -\ 7340 \\ \hline \end{array}$	$\begin{array}{r} 482 \\ -\ 137 \\ \hline \end{array}$	$\begin{array}{r} 4003 \\ -\ 2177 \\ \hline \end{array}$
c.	f.	g.	h.
670 - 97 = _____	$\begin{array}{r} 6004 \\ -\ 1705 \\ \hline \end{array}$	$\begin{array}{r} 7044 \\ -\ 2129 \\ \hline \end{array}$	$\begin{array}{r} 5047 \\ -\ 2036 \\ \hline \end{array}$
i.	j.	'k.	l.
$\begin{array}{r} 1163 \\ -\ 1079 \\ \hline \end{array}$	5400 - 2138 = __________	5216 - 477 = __________	1714 - 311 = __________

ADDITION

(Show all your work in the space provided.)

a.	b.	c.	d.
8 7 3 8 + 9	4 + 7 + 1 + 2 = ______	754 + 567	12 31 91 + 74
e.	**f.**	**g.**	**h.**
6 1 + 0	307 + 48 + 596 + 6 = ______	37 45 + 90	5 + 7 + 0 + 4 = ______
i.	**j.**	**k.**	**l.**
240 886 359 + 794	2 1 2 0 + 1	4 9 6 1 + 1	73 + 59 + 7 = ______

MULTIPLICATION

(Show all your work in the space provided.)

541 x 409	b. 2071 x 368	c. 62 x 4	d. 403 x 59
589 x 7	f. 408 x 9	g. 27 x 104	h. 230 x 2
67 x 60	j. 221 x 4	k. 1203 x 3	l. 313 x 71

DIVISION

(Show all your work in the space provided.)

a. $6\overline{)429}$	b. $4\overline{)1761}$	c. $53\overline{)32134}$	d. $47\overline{)2662}$
e. $27\overline{)1242}$	f. $5\overline{)7005}$	g. $28\overline{)25396}$	h. $9\overline{)4563}$

ADDITION TABLE

+	2	3	4	5	6	7	8	9
2	4	)	6	7	8	9	10	11
3	5	6	7	8	9	10	11	12
4	6	7	8	9	10	11	12	13
5	7	8	9	10	11	12	13	14
6	8	9	10	11	12	13	14	15
7	9	10	11	12	13	14	15	16
8	10	11	12	13	14	15	16	17
9	11	12	13	14	15	16	17	18

MULTIPLICATION TABLE

×	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Appendix V

The University of British Columbia

MATHEMATICS EDUCATION DIAGNOSTIC CENTER

DIAGNOSIS FORM FOR INTERMEDIATE GRADES

Algorithms for Natural Numbers			Comments
Addition	Three-digit nos. with regrouping.	|————|	
	Two-digit nos. with regrouping.	|————|	
	Single column with regrouping.	|————|	
	Single column with no regrouping.	|————|	
	Basic facts.	|————|	
Subtraction	Two consecutive 0's in the minuend.	|————|	
	One 0 in the minuend	|————|	
	With regrouping.	|————|	
	No regrouping.	|————|	
	Basic facts.	|————|	
Multiplication	Three-digit multiplier; incl. zero.	|————|	
	Two-digit multiplier.	|————|	
	One-digit multipler; with regrouping.	|————|	
	One-digit multiplier, no regrouping.	|————|	
	Basic facts.	|————|	
Division	Zero in the quotient.	|————|	

			Comments
Division (cont'd)	Two-digit divisors without or with remainder.		
	One-digit divisor; with remainder.		
	One-digit divisor; no remainder.		
	Basic facts.		
Principles	Commutative addition.		
	Commutative multiplication.		
	Associative addition.		
	Distributive property.		
	Role of 0 in addition.		
	Role of 1 in multiplication.		
	Place value: a) reading and writing nos.		
	b) recognizing "places".		
	c) expanded notation.		
	d) renaming.		
	e) powers of 10.		
	Associative multiplication.		
	10 and its powers as factors.		

3

Comments

Principles (cont'd)	Multiples of powers of 10 as factors.	|————|
	Equal factors.	|————|
Fraction Concepts	Part-whole.	|————|
	Subset-set (equal parts).	|————|
	Subset-set (gen.)	|————|
	Extension to no. line.	|————|
	$\frac{a}{b} = a \div b$.	|————|
	$\frac{a}{b} = a \times \frac{1}{b}$.	|————|
	Order.	|————|
	Equivalence.	|————|
	Names of one.	|————|

Algorithms for Rational Numbers (addition and subtraction)

Subtraction-renaming.	|————|
Addition-renaming.	|————|
Renaming.	|————|
Mixed nos. to improper fractions and vice versa.	|————|
Addition-mixed nos.	|————|
Subtraction-mixed nos.	|————|
Subtraction-proper fractions.	|————|
Addition-proper fractions.	|————|

4

Algorithms for Rational Numbers (cont'd) | Comments

	Subtraction-given LCD	|————|	
	Addition-given LCD.	|————|	

Algorithms for Rational Numbers (multiplication and division)

	Multiplication-mixed numbers.	|————|	
	Multiplication-proper fractions.	|————|	
	Multiplication-whole no. and fraction.	|————|	
	Multiplication-unit fractions.	|————|	
	Reduction property.	|————|	
	Division-mixed nos.	|————|	
	Division-proper fractions.	|————|	
	Reciprocals.	|————|	
Decimal Concepts	Notation.	|————|	
	Place Value.	|————|	
	Number-line.	|————|	

Computation with Decimal Fractions

	Addition.	|————|	
	Subtraction.	|————|	
	Multiplication.	|————|	
	Division-whole no. divisor.	|————|	
	Division-gen.	|————|	
	Change decimal to fraction; vice versa.	|————|	

5

Comments

Percent

Notation.	
% to decimal to fraction.	
Common equivalents.	

References

Asaheld, D. Woodruff, Basic Concepts of Teaching, San Francisco: Chandler Publishing Co., 1962, p. 106.

Brownell, William A. "The Progressive Nature of Learning in Mathematics," The Mathematics Teacher, Vol. XXXVII, No. 4, 1944, pp. 147-157.

Hill, Winfred F., Learning A Survey of Psychological Interpretations, Revised Edition, Toronto: Chandler Publishing Co., 1971, pp. 57-82.

Loree, M. Ray, Psychology of Education, Second Edition, New York: The Ronald Press Company, 1970, p. 557.

Mednic, Sarnoff A., Learning, Toronto: Prentice Hall of Canada Ltd., 1964, pp. 27-53.

Ross, Ramon, "A Description of Twenty Arithmetic Underachievers," The Arithmetic Teacher, April, 1964, pp. 235-41.

Skinner, B.F., The Behavior of Organisms, New York: Appleton-Century-Crofts, Inc., 1938

_______, "The Experimental Analysis of Behavior", American Scientist, 45, 1957, pp. 347-371.

_______, and Charles B. Ferster, Schedules of Reinforcement, New York: Appleton-Century-Crofts, 1957.

_______, "The Science of Learning and the Art of Teaching", Harvard Educational Review, 24, 1954, pp. 86-97.

Sprinthall, Richard, C., Sprinthall, Norman A., Educational Psychology: A Developmental Approach, Don Mills: Addison-Wesley Publishing Co., 1974, pp. 207-218.

Sulzer, Beth,and Roy G. Mayer, Behavior Modification Procedures for School Personnel, Illinois: The Dryden Press, Inc., 1972, pp. 103-133.